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Banneker Banner Submission Guidelines

The Banneker Banner is the official journal of the Maryland Council of Teachers of Mathematics. The journal is named after Benjamin Banneker, a Maryland native and perhaps the first documented African-American mathematician. The Banner is published twice each year and contains a wide range of articles on issues in mathematics education at all levels. Articles published in the journal must be submitted to the editor and undergo a peer review process.

The Banner welcomes submissions from all members of the mathematics education community, not just MCTM members. All submissions should be relevant, interesting, and useful to teachers of mathematics in Maryland. More information about the submission process can be found here.

Reading and Writing in the Mathematics Classroom! What?

Nina Jacks, Prince George's County Public Schools

Abstract: *Recent data demonstrate the need to incorporate reading, writing, and speaking in the mathematics classroom. Teachers often teach the way they were taught, but this does not mean it is always productive. Students need opportunities to discuss, model, and write about their thinking and reasoning. This article gives suggestions on how to bridge the connection of literacy and numeracy in an effort to support students with abstract mathematics concepts.*

Reading and writing do not belong in mathematics. If you agree with this, listen to what some students in a suburban district in Maryland have to say about it:

- "I try to use my reading strategies to understand the math word problems."
- "I hate writing, but my teacher keeps asking me to explain my models. Writing about it makes me think."
- "I used to just try and solve the math problems. My teacher makes us read the information and figure out what the numbers mean."
- "Sometimes I have trouble writing about my math problems. I know how I got the answer, but I can't write it."

For years, teachers have taught reading, writing, and mathematics in isolation; some still do. However, as we shift toward coherence, students need to experience the connections among the three.

Think about it. When you encounter a problem in your everyday life, do you choose strategies in isolation? Do you only read to solve it? Do you only write to solve it? Do you only work out a mathematics problem to solve it? No, of course not! You integrate all necessary methods: You may *discuss* the situation, *read* information to guide you, *write* to someone via email, text, or dare I say, an official letter. You do not function in isolation, so why do we expect students to do so?

Literacy is the bridge to coherence. In my school district teachers use a core tool in developing students' mathematical literacy. This tool, shown in Figure 1, is a double sided document that lists strategies students can use when reading, writing, speaking about, and modeling mathematics.

There are also descriptions of student behaviors in relations to the Standards for Mathematical Practice (SMP); you will notice they are embedded in each quadrant.

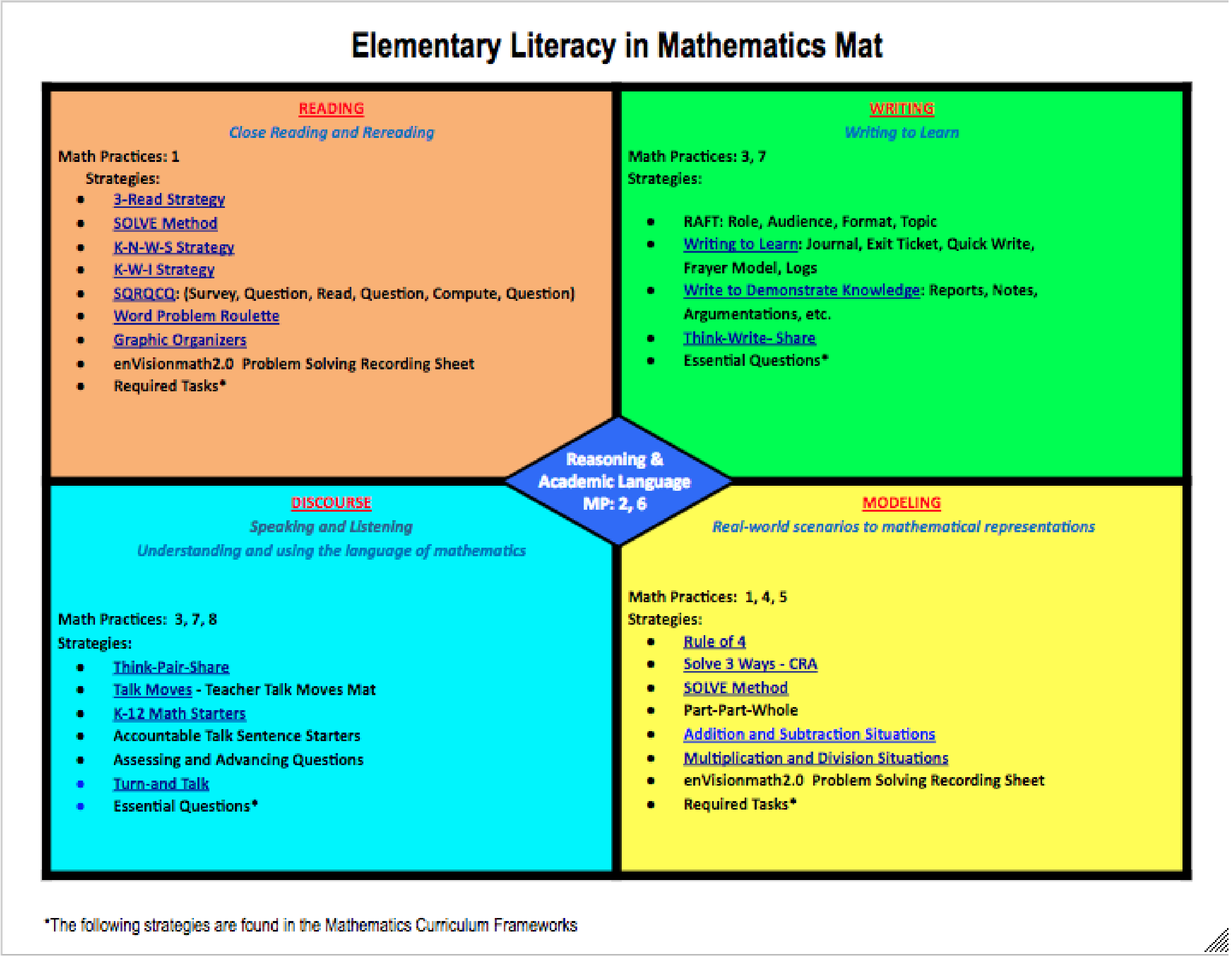

Fig. 1: Elementary Literacy in Mathematics Mat

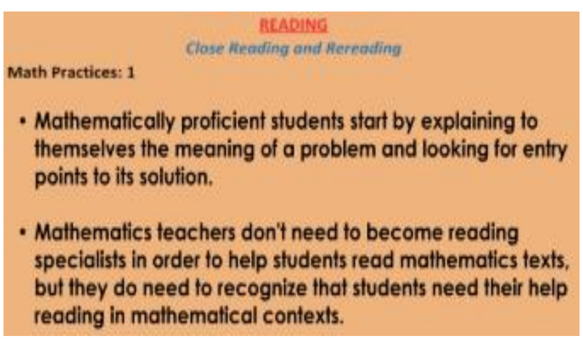

Fig. 2: Standards for Mathematical Practice in Reading

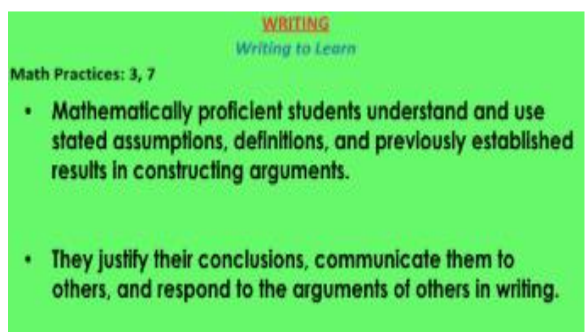

Figure 3: Standards for Mathematical Practice in Writing

This tool is adaptable for middle and high school mathematics classrooms. You will see that students are challenged to process information rather than simply write answers to questions. The following are tips and strategies for adapting this tool and using the SMPs and Universal Design for Learning (UDL) to develop a flexible learning environment that promotes reading and writing in secondary classrooms:

SMP 1-Make sense of problems and persevere in solving them

Middle school teachers could have students use a 3-Read Strategy with the goal of making sense of a multi-step word problem. First, students will read to understand the context. Next, they will read to understand the mathematics by identifying what each numeral represents. Lastly, students will read and annotate by writing comments and questions based on the information presented. High school teachers could have students use Polya's 4 Step Problem Solving Process where students carefully read each part and examine the problem:

1. **<u>Understanding the Problem</u>** What is known? What are the data? What is the condition? Can I draw a visual model?
2. **<u>Devising a Plan</u>** Have I seen this problem or a similar one before? Is there a useful theorem? What is the connection between data and the unknown? Can I solve part of this problem? How can I use the methods and results from a related problem that I previously solved? What plan do I have for a solution?
3. **<u>Carrying out the Plan</u>** Is each step correct? Can I prove that it is correct? Did I use all the data? Did I use the whole condition?
4. **<u>Looking Back</u>** Can I check the results? Can I check the argument? Can I derive the result differently? Can I see it at a glance? Can I use this result or method for another problem?

SMP 7- Look for and make use of structure

Let us examine how middle school teachers could have students write for different purposes using **R**ole **A**udience **F**ormat **T**opic (R.A.F.T). In one example, a student could take on the role of the digit zero. Zero (R) writes a speech (F) addressing Mr. Number Line or Mrs. Place Value (A) explaining why Zero is important and should always be considered when other digits get together (T).

SMP 3-Construct viable arguments and critique the reasoning of others

High school teachers can set up a debate that would encourage discussion and argumentation around topics and concepts where students have to write what they know about shapes, lines, and angles, and then read and critique the reasoning of their peers using proofs in a geometry class.

One strategy for all grade levels is to have students annotate the task. This helps to organize ideas, thoughts, questions, and misconceptions, while promoting thinking. Take a page from UDL and encourage students to make the mathematics meaningful by using what is learned in multiple ways and apply their understanding of concepts orally, visually, or in a tactile and kinesthetic manner. Additionally, students should have opportunities to customize their display of knowledge with graphic organizers and templates when collecting data or organizing information. Offer students options for sustaining effort and persistence by providing frequent feedback that will motivate students to make corrections in real time as learning is taking place. The feedback should emphasize student effort, improvement, and achievement.

Students demonstrate understanding by being active participants in their learning as they engage in creating visual models and make connections to the abstract to solidify their ideas. They pose and respond to questions. Even if students' questions are not answered during a discussion, it gets them thinking and problem solving. In turn, students develop new pathways to learning.

What Do the Data Say?

In an effort to make cross-curricular connections, educators must aid students in understanding the interrelatedness of reading, writing, speaking, and mathematics. Students' vocabulary development and writing to justify their thinking and reasoning are key components to sharing understanding of mathematics concepts. Data from the 2017 Partnership for Assessment of Readiness for College and Careers (PARCC) reported low averages of students in Maryland who met or exceeded expectations for mathematics: (43% Grade 3; 37.5% Grade 4; 35.5% Grade 3; 32.2% Grade 6; 25.4% Grade 7; and 16.8% Grade 8). The PARCC test required students to read for understanding on 39% of the questions and write to justify their reasoning on 21% of the questions.

Research

As educators, we must provide opportunities for students to read, write, and talk about the mathematics they are learning. If students are not sharing their own thinking and reasoning, proving their answers, and critiquing the arguments of others, they are not fully accessing learning. Reinhart's (2000) study concluded:

> To help students engage in real learning, I must ask good questions, allow students to struggle, and place the responsibility for learning directly on their shoulders. I am convinced that children learn in more ways than I know how to teach. By listening to them, I not only give them the opportunity to develop deep understanding but also am able to develop true insights into what they know and how they think. (p.483)

When students engage in tasks that promote reasoning and problem solving, they must read for understanding, discuss their thinking, create a visual model of their ideas, and write to justify why their solution is correct. Through this problem solving process, students are interacting with multiple forms of the same information at the same time. Boaler (2013) concluded that ability and intelligence grow with effort and practice.

In closing, I ask once again that you consider how *you* solve real word problems. What do you think about when faced with a challenge? What strategies do you employ? Students need to engage in learning with rich tasks that promote thinking and reasoning. Teachers must facilitate this learning with questions that require high cognitive demand so students can grow their intelligence. Do reading and writing belong in the mathematics classroom? Reading, modeling, talking and, yes, writing about mathematics aid in solidifying concepts and challenge students to express their learning in multiple forms. Students will cross the bridge to coherence, if teachers set them on the path of literacy.

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If Math were an Animal: Uncovering Productive and Unproductive Beliefs and Attitudes about Mathematics

Karen Zwanch, Virginia Tech

Abstract: *A writing prompt was used to gauge elementary students' beliefs about mathematics, and analysis revealed five distinct categories. Each category is defined, example responses are given, and responses are related to (un)productive beliefs about mathematics. Implications are given to help teachers apply this prompt and interpret their students' beliefs.*

The National Council of Teachers of Mathematics (NCTM, 2014) identifies productive and unproductive beliefs about mathematics teaching and learning that support or impede students' learning. Conceptualizing beliefs in this way can help teachers understand and act upon their students' beliefs. Therefore, this article describes how a writing prompt used in elementary mathematics classrooms can help teachers uncover and identify their students' beliefs and determine whether those beliefs are productive or unproductive.

Productive and Unproductive Beliefs in the Classroom

It is important for teachers to understand how students' beliefs impact learning, how instruction can impact students' beliefs, and to have tools for measuring students' beliefs about mathematics. The importance of understanding how students' beliefs impact learning is highlighted in the *Principles to Actions* (NCTM, 2014). "Students' beliefs influence their perception of what it means to learn mathematics and their dispositions toward the subject" (p. 11). Students' conceptions of mathematics may vary from the belief that mathematics is about memorizing facts, to mathematics being a way of thinking, to name just two. Students' may also hold productive or unproductive beliefs about mathematical ability. A fixed mindset implies the belief that mathematical ability is predetermined (Dweck, 2008); a student who is bad at mathematics cannot improve and conversely, a student who is good at mathematics will always be a good mathematics student. A more productive belief is a growth mindset, which is characterized by a student's belief that he/she can learn mathematics through persistence (Boaler, 2016). These beliefs are two examples of the types of beliefs that students may hold, and how these beliefs may vary.

Furthermore, these beliefs effect how and to what extent students participate in mathematical activity. For example, the belief that mathematics is about memorizing facts may contribute to students' unwillingness to engage in problem solving if they do not perceive it to contribute to their learning. Alternatively, the belief that mathematics is a way of thinking may contribute to students' willingness to persist with real world applications of mathematics. These differences in engagement contribute to the quality of each student's learning.

Instruction is an important factor that influences students' beliefs and attitudes about mathematics as they progress through school (Schoenfeld, 2011). Regardless of whether students hold productive or unproductive beliefs, both their attitudes (related to their feelings) and their beliefs (related to cognition) are malleable (Beswick, 2006). Thus, it is the teacher's responsibility to understand how instruction impacts the mathematical beliefs of their students.

Some unproductive beliefs may result from well-intentioned instruction. For example, the beliefs that mathematics is about memorization or that problems should be solved quickly (Schoenfeld, 1989, 2011) are harmful to students' learning if, for instance, students quickly give up because they decide a problem is impossible or if students indiscriminately apply rote procedures to problems. Teachers can shape their students' mathematical experiences to foster more productive beliefs and work to avoid practices that lead to unproductive beliefs.

A fixed mindset is another example of an unproductive belief that can stem from well-intentioned instruction. Presenting step-by-step solutions, thus eliminating productive struggle (NCTM, 2014), may lead to the belief that mathematics should be a smooth and linear process. In reality, mathematics is more of a cyclical process of problem solving that is challenging at times. Teachers' instructional strategies may inform which of these beliefs students hold. Shifting instructional focus away from right answers is one powerful way to address a fixed mindset (Boaler, 2016). Teachers can also praise mistakes as learning opportunities, encourage learning through challenges (Boaler, 2016), encourage persistence, team work, conjecturing, and long-term problem solving (Clarke and Clarke, 2003). Each of these strategies has been shown to support students' productive beliefs about mathematics. This demonstrates the importance of teachers' instructional decision making in the formation of students' beliefs.

Teachers can also influence students' beliefs about mathematical activity and what mathematics is. Presenting mathematics as a "cultural phenomenon: a set of ideas, connections, and relationships that we can use to make sense of the world" (Boaler, 2016, p. 23) helps students view mathematics as a way of thinking instead of a school activity. Planning lessons to focus first on conceptual understanding will support more productive beliefs (NCTM, 2014) and improve students' attitudes and achievement (Boaler, 2016). Beginning instruction with problem solving and critical thinking helps students understand that mathematics is imbued in daily activity.

Principles to Actions (NCTM, 2014) gives teachers substantial guidance in supporting productive beliefs, through considerations such as encouraging problem solving, stimulating discussion, connecting mathematical representations, building conceptual understanding, and encouraging perseverance. Each of these strategies can be applied at any grade level or with any content standards to promote productive beliefs, but the teacher must devise methods to measure students' beliefs. This article presents a writing prompt used in elementary classrooms that accomplishes such a goal, as well as guidelines for interpreting students' responses to the writing prompt and whether these responses imply productive or unproductive beliefs about mathematics.

Uncovering Students' Productive and Unproductive Beliefs

The writing prompt, "If math were an animal, it would be a _____ because _____," was given to 48 elementary students by two teachers. The first, a fourth grade classroom teacher, gave the prompt to 17 students. The second, a Title I mathematics teacher, gave the prompt to 31 students from second and fifth grades.

From the 48 responses, five categories emerged: *Topics in Mathematics*, *Attitudes about Mathematics*, *Who can Learn Mathematics?*, *What is Mathematics?*, and *How do we do Mathematics?* (Table 1). A few students' responses fit squarely within only one category, although most students' responses were lengthy, and were included in more than one category. Responses were not categorized based on the animals students chose. Rather, responses were categorized based on students' reasons for selecting the animals. For example, indicating mathematics is like a monkey could mean different things – a monkey is smart, can learn tricks, or is fun – and indicate different beliefs. Additionally, responses were coded as positive if they indicated a productive belief, negative if they indicated an unproductive belief, or neutral if they did not indicate a productive or unproductive belief. In the next sections, each category is explained, the types of responses that fit within each category are defined, and the responses are related to productive and unproductive beliefs about mathematics (NCTM, 2014).

Table 1. Categories, Definitions, and Examples from Students' Responses to the writing prompt, "If Math Were an Animal, it would be a _____ because ______."

Category: Definition
- Examples

Topics in Mathematics: Students related an animal to a math concept, manipulative, symbol, or to a math vocabulary word.
- Adding an animal's legs (e.g., spiders)
- Similarities between animals and math symbols (e.g., an alligator and inequality symbols)

Attitudes about Mathematics: Students' responses indicated either a positive or negative affect toward math.
- Hate
- Mean
- Attacking
- Love
- Awesome
- Fun

Who can Learn Mathematics?: Students explained characteristics of their chosen animal as it relates to the animal's aptitude for math.
- Smart animals
- Hard working animals

What is Mathematics?: Students explained characteristics of their chosen animal as it relates to the nature of math, including its usefulness.

- Waste of time
- Lots to know
- Useful
- Helpful

How do we do Mathematics?: Students' responses gave insight into what they believe it means to engage in mathematical activity.

- Stay calm
- Be trained
- Do worksheets
- Work hard

Topics in Mathematics

Responses in this category related topics in mathematics to animals. In these responses, students: (a) described physical characteristics of an animal that look like a mathematics symbol or manipulative (e.g., an alligator's mouth and an inequality symbol); (b) described applying a mathematics concept to an animal's physical appearance or behavior (e.g., a piranha eating the bigger number in an inequality problem); (c) compared the size of an animal to the size of a number (e.g., elephants to big numbers); or (d) listed mathematics vocabulary words.

There were 13 responses in this category, and all were coded as neutral because they did not indicate a belief. These responses may not help teachers understand students' beliefs, but encouraging and structuring writing activities in the mathematics classroom will help students become more thorough in communicating their thinking (Casa, 2015/2016; Williams & Casa, 2011/2012). Thus, even if students' responses to this prompt are initially not very insightful, persistence can lead them to more thoroughly communicating through writing.

Attitudes about Mathematics

The second category illuminated students' attitudes about mathematics, which were highly polarized; nine students wrote about loving mathematics and seven wrote about hating mathematics. Responses that indicated feelings or affect toward mathematics were included in this category. Students who said they love mathematics, that mathematics is awesome, or that compared mathematics to their favorite animal were coded as indicating a positive attitude. One student likened mathematics to a cat because "…I love math and I love cats." Responses were coded as indicating a negative attitude toward mathematics if students described mathematics as hurting, annoying, or mean. One student explained "when I'm in school it [math] hunts me down, then it finds me and attacks me!" Allowing students to verbalize their attitudes can incite discussions about students' feelings toward mathematics.

Who can Learn Mathematics?

This category included responses from 16 students, and responses described an animal's ability to learn mathematics. Thirteen responses listed an animal the student felt could learn mathematics because it is smart. The implication is that some students are simply born dolphins and monkeys—they can learn mathematics—while other students cannot. One student explained mathematics is like "an ape because you half to know a lot of math and they do know a lot." The latter half of this rationale, that "they do know a lot," implies a static state of knowledge rather than an incremental state of learning, which indicates an unproductive belief. Another student went as far as to state some people can understand math while others will just become frustrated by it. The belief that students will either understand or not understand mathematics leads them to limit their effort because they believe the outcome is predetermined (Dweck, 2008). These responses indicate a fixed mindset toward learning mathematics, which is an unproductive belief.

An indication that you must be smart to learn mathematics was the most common response in this category, but three responses indicated a mathematical mindset (Boaler, 2016). These responses explained an animal's ability to learn mathematics by working hard. One student explained mathematics is like a deer because "the deers started on their work and the deer said I've study really hard and at the end he got a A+." This is a more productive belief because it implies achievement can be improved through effort.

What is Mathematics?

Beliefs about the nature of mathematics relate to what mathematics is and why it is important. Productive beliefs are that mathematics is a coherent subject based in problem solving and reasoning, whereas unproductive beliefs are that mathematics is about memorization and procedures (NCTM, 2014). Seven responses fit within this category and five responses indicated an unproductive belief. Responses included students indicating that: (a) in mathematics there is lots to know; or (b) mathematics is useful or not useful. The first subgroup of responses indicates mathematics is about learning disconnected facts and procedures. One student explained in mathematics there are "lots of details" to learn. Another compared mathematics to a dinosaur because "Dinosores are big and it could fit all the math facts on." This imagery of a dinosaur covered in mathematics facts speaks to the student's belief that mathematics is about learning facts. While mathematics is certainly broad, students are explaining that they view mathematics as disjointed. This belief causes students to view mathematics as unattainable, not worthwhile, or repetitive. Students indicating mathematics is not useful similarly believe that mathematics is an unrelated set of facts. This belief contributes to a fixed mindset and aligns with an unproductive belief. On the other hand, students indicating mathematics is useful or helpful indicates mathematics is a way of thinking, which is a more productive belief.

How do we do Mathematics?

The final category revealed students' beliefs about mathematical activity. These 13 responses characterized doing mathematics as: (a) students' passive or active roles in mathematical activity, such as being trained (passive) or working hard (active); or (b) activities students perceive to be representative of mathematical behavior. One student compared mathematics to a dog because to do mathematics, students must be trained, and dogs are easily trained. Students' beliefs about mathematical activity may stem from their experiences in mathematics class (Schoenfeld, 1989, 2011), so focusing a majority of time passively learning leads to unproductive beliefs about doing mathematics.

Some responses in this category were difficult to categorize as indicating productive or unproductive beliefs because passive acts are not necessarily bad and active acts are not necessarily good. However, mentions of doing worksheets or playing games indicates these activities are pervasive in the students' classrooms, and their pervasiveness influences beliefs about mathematical activity. Four responses indicated productive beliefs, like hard work and working together, six indicated unproductive beliefs, like being trained or bossed, and three were coded as neutral. Table 2 summarizes students' responses that are characteristic of each of the categories described.

Table 2. Polarized, Representative Student Responses from each Category

Topics in Mathematics

- Neutral: "If math was an animal, it would be an anteater because the nose looks like a subtraction sign." -4th grader
- Neutral: "If math were an animal it would be a cheta because of the spots. You can count the spots." -2nd grader

Attitudes about Mathematics

- Positive Attitude: "If math was an animal it would be a snake because it is awesome like math." -4th grader
- Negative Attitude: "If math were an animal it would be a Pittbull because it bites you in the Rear end" -4th grader

Who can Learn Mathematics?

- Productive Belief: "If math were an animal it would be a Deer because if the deer went to school … and said i'll study really hard and get an A+" -5th grader
- Unproductive Belief: "If math was an animal, it would be a dolphin because dolphins are smart." -4th grader

What is Mathematics?

- Productive Belief: "If math were an animal it would be a dog a German Shepard because … they are helpful with they're smell they are police dogs" -5th grader
- Unproductive Belief: "If math were an animal it would be a dinosore because dinosores are big and it could fit all the math facts on it." -2nd grader

How do we do Mathematics?

- Productive Belief: "If math was an animal, it would be a lion becaus lions work hard and we do in math." -4th grader
- Unproductive Belief: "If math were an animal it would be a dog because I like dogs. They are easy to train." -5th grader

Successfully Gauging Students' Beliefs

The responses in this article are given as examples to guide teachers in understanding their own students' beliefs and attitudes about mathematics through the use of this writing prompt, however, categorizing students' responses ultimately depends upon teachers' professional judgement. The following are suggestions to support these judgments. First, it is important to think about the animal *and* the student's rationale for selecting the animal. For example, six students selected a dog, but the students' rationales varied. Some students explained that mathematics is a dog because dogs are their favorite animal or that dogs are useful, while other students responded that dogs bite. Of course, these responses indicate very different things, demonstrating the importance of understanding the student's rationale.

Writing regularly in mathematics class, making connections between discussion and writing (Casa, 2015/2016), and scaffolding writing (Williams and Casa, 2011/2012) help elementary mathematics students become more successful writers. If students' responses to this prompt are very brief, teachers should encourage students to expand. Teachers who implemented the writing prompt in this study did not ask students to elaborate, but in retrospect, that may have been helpful in interpreting some of the responses. Asking students to expand or implementing writing strategies will improve the insight teachers can gain into their students' beliefs.

Finally, teachers are encouraged to collaborate with others when categorizing and interpreting students' beliefs. Not only will this provide teachers with deeper assurance that they are accurately interpreting students' beliefs, but it will also allow multiple teachers to develop deeper understanding of each child's beliefs. Asking students whether they view mathematics as a "hard working lion" or an "attacking shark" will facilitate conversations about students' feelings about mathematics and their beliefs about learning and doing mathematics, while additionally providing teachers with feedback on whether their instruction is supporting productive or unproductive beliefs.

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Mathematical Soup

Robert Koca, Community College of Baltimore County

Abstract: *Finding least common multiples and greatest common factors are related to logic questions involving soup recipes. The ingredients lists for the recipes have a connection to prime factorizations of numbers. This gives students who can answer the logic questions the ability to then find least common multiples and greatest common factors by using prime factorizations.*

Overview

A method of teaching some basic ideas from number theory including how to find least common multiples and greatest common factors is given. The approach is first asking logic questions related to soup recipes where the ingredients for the recipes have a similar structure to the prime factorizations of numbers and then showing how those logic questions are essentially the same idea as the number theory concepts. This activity is suitable for students who already know what prime numbers are, know how to find a prime factorization with the factor tree method, know the concepts of common multiples and common factors, and can find common multiples and common factors for small values by the listing method.

A suggested lesson plan is given consisting of reading to be done by the students with problems to be done. Solutions for those problems are given afterwards. Further problems are based on understanding the previous material. Thus, leading the lesson by having the students read the material and answer a few of the questions and then going over the solutions to those questions before continuing on to later questions is suggested. Some students though could go through all the questions on their own. The solutions also include comments to make or suggestions on additional examples to do to lead into the next questions. If given as a handout the instructor could choose to give just the reading and problems or could decide to include the solutions as well.

Mathematical Soup

A soup chef has many recipes labeled in a book by number and ingredients used. Some soups like soup 2 or soup 13 are very simple and are just water with a single other ingredient. Other soups such as soup 4 use two servings of some ingredient with the water and others such as soup 30 use many different ingredients. The instruction for each recipe is to just mix all the ingredients together with water and heat. Since every soup in this recipe book uses Water it is assumed as an ingredient for each of the soups. The recipe book starts as:

Soup 1: Just Water

Soup 2: A serving of Asparagus

Soup 3: A serving of Broccoli

Soup 4: Two servings of Asparagus

Soup 5: A serving of Cauliflower

Soup 6: A serving of Asparagus and a serving of Broccoli

Soup 7: A serving of Dill

Soup 8: Three servings of Asparagus

Soup 9: Two servings of Broccoli

Soup 10: A serving of Asparagus and a serving of Cauliflower

Soup 11: A serving of Eggplant

Soup 12: Two servings of Asparagus and a serving of Broccoli

Soup 13: A serving of Fennel

Soup 14: A serving of Asparagus and a serving of Dill

Soup 15: A serving of Broccoli and a serving of Cauliflower

Soup 16: Four servings of Asparagus

Soup 17: A serving of Garlic

The more fancy recipes using many ingredients will be the later recipes with higher numbers. It may not appear so at first glance but these recipes are numbered in a very orderly way. Do the first two problems yourself (the effort here is just as important as getting the correct answer, take at least a minute of think time on each) and then wait for your teacher to go over the answers. While waiting you can try to guess what the next recipes are and then can read on to check.

Problem 1: Which soups use just a single serving of a single ingredient (in addition to Water)? What do all those numbers have in common? What would be the next soup using a single ingredient?

Problem 2: Soup 15 uses the ingredients found in soup 3 and soup 5. Using this as an example what is a reasonable guess for the ingredients for soup 35? What is a reasonable guess for the ingredients for soup 44?

Here is the next page of the recipe book:

Soup 18: A serving of Asparagus and two servings of Broccoli

Soup 19: A serving of Ham

Soup 20: Two servings of Asparagus and a serving of Cauliflower

Soup 21: A serving of Broccoli and a serving of Dill

Soup 22: A serving of Asparagus and a serving of Eggplant

Soup 23: A serving of Impastata (a kind of cheese)

Soup 24: Three servings of Asparagus and a serving of Broccoli

Soup 25: Two servings of Cauliflower

Soup 26: A serving of Asparagus and a serving of Fennel

Soup 27: Three servings of Broccoli

Soup 28: Two servings of Asparagus and a serving of Dill

Soup 29: A serving of Jackfruit

Soup 30: A serving of Asparagus, a serving of Broccoli, and a serving of Cauliflower.

Now try problems 3 and 4 and then wait for your teacher to go over the answers. While waiting you can try the same questions for some different soups.

Problem 3: The chef now has the ingredients to make soup 30, namely a serving of Asparagus, a serving of Broccoli, and a serving of Cauliflower (and also Water). From that list of 30 soups what soups could be made with those ingredients? For example the chef could make a bowl of soup 5 since the ingredient of Cauliflower is indeed available. Soup 14 could not be made though. The serving of Asparagus is available but the necessary serving of Dill needed for soup 14 is not available; only Asparagus, Broccoli, Cauliflower and Water are available.

Problem 4: Now the chef has the ingredients to make soup 24, namely three servings of Asparagus and a serving of Broccoli (and Water). What soups could be made with those ingredients?

Now try problems 5 and 6 and then wait for your teacher to go over the answers. While waiting you can think about what math concepts are being shown by these soup problems.

Problem 5: Suppose the chef only has water available but no other ingredients. A friend is coming over for dinner and to know what ingredients to go buy the chef asked the friend what soup the friend would like. The friend says, "Either soup 6 or 10, I will let you know when I arrive at your house". What does the chef need to buy to ensure that the soup the friend chooses could be made? Is there an ingredient or ingredients that will definitely be in the soup that the chef makes?

Problem 6: Same as question 5 but instead the friend said "Either soup 4 or 5, I will let you know when I arrive at your house."

You may think you were just doing some logic puzzles but you have actually just learned some math concepts. If you have the prime factorizations of two numbers (what you get from a fully done factor tree) how you find a least common multiple (LCM) is like the problem of the chef deciding what ingredients are needed in order to be ready to make either of two soups. How you find a greatest common factor (GCF) is like the problem of what ingredients will definitely be used. From question 5 we saw that the chef needed to buy the ingredients which are needed for recipe 30. 30 is the least common multiple of 6 and 10. We also saw that the ingredients for soup 2 would definitely be used and 2 is the greatest common factor of 6 and 10. From question 6 we saw that that the chef needed to buy the ingredients which are needed for recipe 20. 20 is the least common multiple of 4 and 5. 20 is the least common multiple of 4 and 5. We also saw that the ingredients for soup 1 would definitely be used and 1 is the greatest common factor of 4 and 5.

You probably could have found LCM(6, 10) and the other questions by the listing method. For example, to find the least common multiple of 6 and 10 one can list the multiples of 6: 6, 12, 18, 24, 30, 36, … and of 10: 10, 20, 30, 40, … and see that 30 is the first value in both lists. That technique works well for small numbers but what if the numbers are larger? For example, suppose you are finding the GCF and LCM of 7875 and 1485? Making the lists of multiples or factors is more difficult and it might take a while to find the first value that is in both lists for finding the LCM or the largest value that is in both lists for the GCF.

A way to think of the prime factorization of a number is that it gives the numerical ingredients needed to make that number. Instead of mixing soup ingredients the numerical ingredients are multiplied. Note that mix and multiply both begin with the letter m. The GCF of two numbers are the common ingredients and the LCM of two numbers is the smallest amount of ingredients needed to make either number. As an example let's use this approach with 7875 and 1485. The prime factorizations are $7875 = 3^2 \times 5^3 \times 7$ and $1485 = 3^3 \times 5 \times 11$.

The numerical ingredients for 7875 are 2 servings of 3, three servings of 5, and a single serving of 7. Those ingredients are multiplied to give 7875. The numerical ingredients for 1485 are three servings of 3, a single serving of 5 and a single serving of 11. They are multiplied to give 7875.

To be ready to make either of these numbers we need to have on hand three 3's, three 5's, a single 7 and a single 11. Thus, $\text{LCM}(7875, 1485) = 3^3 \times 5^3 \times 7 \times 11 = 259875$.

If one of those number is made the common ingredients will definitely be used. These are two 3's and a single 5. Thus, $\text{GCF}(7875, 1485) = 3^2 \times 5 = 45$. It will take a little bit of time to find the prime factorizations but for larger numbers this method is much faster than the listing method. If you want to test that for yourself you can try to find LCM(7875, 1485) by using the listing method.

Here are a couple more problems to test yourself on. Some hints are that the prime factorizations are $12 = 2^2 \times 3$, $16 = 2^4$, $56 = 2^3 \times 7$, and $489804 = 2^2 \times 3 \times 7^4 \times 17$.

While waiting for your teacher to go over the answers you can think about what questions about the soup recipes would match these math questions.

Problem 7: A) Find GCF(12, 16) B) Find LCM(12, 16)

Problem 8: A) Find GCF(56, 489804) B) Find LCM(56, 489804)

Prime factorizations can also help one decide if one number is a factor of another. If I asked you "Is 3 a factor of 12?" you would say yes because you know that 3 x 4 = 12. Another way to know that 3 is a factor of 12 is to see that the prime factorization of 12 is 2^2 x 3 and 3 appears in that prime factorization. A matching soup question is "Are the ingredients for 3 ALL used in the ingredients for soup 12?" The answer is yes; all the ingredients (just Broccoli and Water) are ingredients for soup 12.

Here is another example with the numbers being a little larger. Is 14 a factor of 84? For this example, it is possible to just divide and since 84 divided by 14 gives a quotient of 6 with no remainder we know that 14 is a factor of 84. Let's do it with the prime factorizations technique to see the idea. First find the prime factorizations of 14 = 2 x 7 and 84 = 2^2 x 3 x 7. All of the ingredients for 14 are also used in 84 so 14 is a factor of 84.

Here is one final problem to test this last concept. Note that since the prime factorizations are given there is no need at all to do a division. No calculations are needed.

Problem 9: Without doing a full division determine if
a) 6 is a factor of 1503378.

b) 21 is a factor of 1503378.

HINTS: The prime factorizations are:

$6 = 2 \times 3$

$21 = 3 \times 7$

$1503378 = 2 \times 3^2 \times 17^4$

SOLUTIONS:

Problem 1: Soups 2, 3, 5, 7, 11, 13, and 17 all use a single ingredient. Those are all prime numbers. If that pattern continues, soup 19 would be the next soup to use a single ingredient.

Problem 2**:** The key observation is that 35 = 5 x 7. Following the patterns, a reasonable guess is that soup 35 uses the ingredients for soups 5 and 7. So soup 35 uses a serving of Cauliflower and a serving of Dill. Since 44 = 4 x 11 it would use the ingredients for soups 4 and 11, namely, two servings of Asparagus and a serving of Eggplant. Alternatively, 44 = 2 x 2 x 11 and thus uses the ingredients for soup 2 twice and for soup 11 once also giving the answer of two servings of Asparagus and a serving of Eggplant.

Those patterns are actually how the recipe book is organized. Any soup with a prime number is just a single ingredient (with the Water) and the ingredients for the composite numbers can be found by breaking down the number. As a few more examples for soup 18 we should think that 18 = 2 x 3 x 3 and thus uses the ingredients for soup 2 and the ingredient for soup 3 twice, namely a serving of Asparagus and two servings of Broccoli. Since 19 is prime, soup 19 is a single ingredient soup. Another pattern is that each new ingredient starts with the next letter. So far the ingredients are Asparagus, Broccoli, Cauliflower, Dill, Eggplant, Fennel, and Garlic.

It is worth pointing out that multiplication is the key idea and not addition. For example if a soup is made using the ingredients Cauliflower (soup 5) and Eggplant (soup 11) its number would be 11 x 5 = 55 and not 5 + 11 = 16.

Problem 3: The chef could make soups 1, 2, 3, 5, 6, 10, 15, and 30. Let's look at a few soups that are not possible. Soup 4 is not possible because that would require two servings of Asparagus. Soup 21 is not possible because that would require having a serving of Dill.

Problem 4: The chef could make soups 1, 2, 3, 4, 6, 8, 12, and 24. Note that soup 16 is not possible since that would require four servings of Asparagus but only three servings are available.

Problem 5: The chef will need a serving of Asparagus, a serving of Broccoli, and a serving of Cauliflower. Note that only a single serving of Asparagus is needed. The chef is able to make one or the other of the soups and does not need to make each. Also note that those ingredients are the same as needed for soup 30. Whichever soup is made, a serving of Asparagus will definitely be used. It is a common ingredient to soup 6 and soup 10.

Problem 6: The chef will need two servings of Asparagus and a serving of Cauliflower. We can only say that Water will definitely be used.

The next section has some information for the student to read before answering the next questions. Instructors may want to give a few additional examples of this type and may want to also include an example where one first finds the prime factorizations using factor trees. Since it comes up in the next problem you might want to verify using a factor tree that the prime factorization of 489804 is 2^2 x 3 x 7^4 x 17.

Problem 7: A) GCF(12, 16) = 2^2 = 4. The matching soup scenario is that if the chef makes either soup 12 or 16 that 2 servings of Asparagus will definitely be used.

B) LCM(12, 16) = 2^4 x 3 = 48. The matching soup scenario is that to be ready to make either of soup 12 or 16 the chef needs 4 servings of Asparagus and 1 serving of Broccoli. Those ingredients match what is needed for soup 48 in the cookbook.

Problem 8: A) GCF(56, 489804) = 2^2 x 7 = 28. The matching soup scenario is that if the chef makes either soup 56 (ingredients of 3 servings of Asparagus and 1 serving of Dill) or soup 489804 (ingredients of 2 servings of Asparagus, 1 serving of Broccoli, 4 servings of Dill, and 1 serving of Garlic) that 2 servings of Asparagus and 1 serving of Dill will definitely be used. Those ingredients match soup 28 in the cookbook. B) LCM(56, 489804) = 2^3 x 3 x 7^4 x 17 = 979608. The matching soup scenario is that to be ready to make either of soup 56 or 489804 the chef needs to have 3 servings of Asparagus, 1 serving of Broccoli, 4 servings of Dill and 1 serving of Garlic. Those ingredients match what is needed for soup 979608 in the cookbook.

Problem 9: Since a 2 and a 3 are in the prime factorization of 1503378 we know that 6 is a factor of 1503378. Since there is no 7 in the prime factorization of 15033378 we know that 21 is not a factor of 1503378. As a soup question this is just saying that if you have all the ingredients in your pantry to make soup 1503378 which are a serving of Asparagus, two servings of Broccoli and four servings of Garlic that you definitely could make soup 6 but you could not make soup 21 (you do not have any Dill).

Further examples of these types of questions could now be given. One extension is to find the LCM or GCF of three or more values. Another is finding the LCM or GCF as part of a larger application problem.

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The 100 Bead String: A Tool You Can Count On (and More!)

Kristen Mangus, Howard County Public School System

Abstract: *The 100 bead string is a versatile tool that can be used to develop fluency in 1st through 5th grade mathematics classes. Skills that can be developed are place value, number identification, subitizing, rounding, addition, subtraction and multi-digit multiplication with whole numbers and decimals. The 100 bead string also can be used to make connections among representations including number lines and equations. Additionally, the 100 bead string is the perfect tool for helping students to make sense of the Standards for Mathematical Practice while building their conceptual understanding of a variety of mathematics topics.*

"Fluency builds from initial explorations and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems" (NCTM, 2014, p. 42). Math teachers have a variety of tools available to help students develop fluency in their classrooms. Some tools have limited use while other tools span multiple grade levels and standards. The 100 bead string is a versatile tool that can be used in first through fifth grades for a wide range of mathematics standards. This tool helps students bridge from the concrete to the abstract as well as make connections between representations.

Place Value

In first and second grade students work with place value for numbers up to 100. Students can use the 100 bead string to make a number as well as identify the tens and ones by counting the alternating groups of ten by color and then the leftover ones. Teachers can also show a 100 bead string and ask students, "What number do you see?" and "How do you know?" to promote a discussion of place value.

Begin with multiples of ten:

What number do you see? (30) How do you know (I counted 3 groups of ten.)

Then as students become proficient with identifying multiples of ten, show numbers with tens and ones:

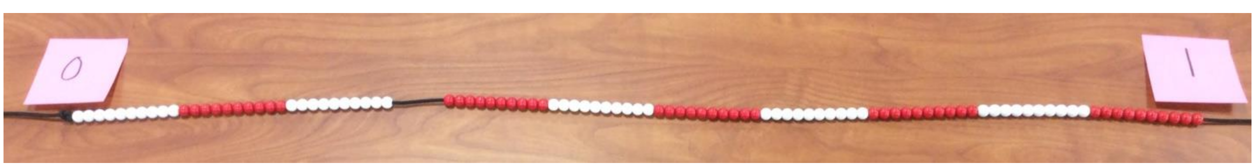
What number do you see? (62) How do you know? (I saw 3 red groups of ten and 3 white groups of ten for a total of 60 and then 2 more ones.)

After students demonstrate mastery of counting groups of tens and ones, this can become a number routine at the beginning of math class to promote recognizing numbers up to 100. Simply flash a 100 bead string or image of a 100 bead string for a few seconds, and then ask students, "What number do you see?" and "How do you know?"

Now fast forward to fifth grade. The 100 bead string can also be used to develop place value skills with decimals! Start by designating the endpoints as 0 and 10 (or any other decade) and allow students to discover that the individual beads are tenths and each interval of red or white beads represents a whole number. Now have students identify the decimal and explain how they know.

What number do you see? (7.4) How do you know? (I see 7 groups of ones which is 7 wholes and 4 leftover tenths.)

Or change the endpoints to 0 and 1, and allow students to discover that each individual bead now represents hundredths and each interval of ten red or white beads represents tenths. Continue to ask students, "What number do you see?" and "How do you know?"



What number do you see? (0.3). How do you know? (I see 3 groups of one tenth.)

Next, have students explore decimals greater than one by changing the endpoints to 4.0 and 5.0 or 76.0 and 77.0 or any other interval of one. Again, ask students, "What number do you see?" and "How do you know?"

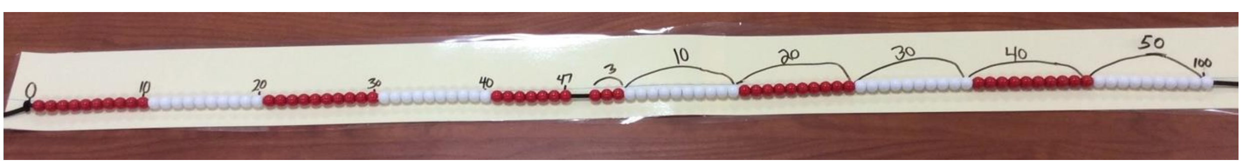
What number do you see? (4.38) How you do know? (I see 3 groups of one tenth and 8 more hundredths.)

Just like with whole numbers, as students demonstrate mastery with identifying decimals on the 100 bead string, this can be used as a number routine at the beginning of math class to promote recognizing decimals, by flashing a 100 bead string or image of a 100 bead string, and then ask students, “What number do you see?” and “How do you know?”

After students become comfortable with identifying either whole numbers or decimals using the 100 bead string, students can show a given number on the 100 bead string, and then tell how far away the given number is from one or both of the endpoints.

For example, in first and second grade, students can use the strategy of counting up to subtract numbers from one hundred. Show a number with the 100 bead string and hide or cover up the remaining beads. Then ask how many more to 100? Students then use the 100 bead string to make the number and then discover how many more to 100.

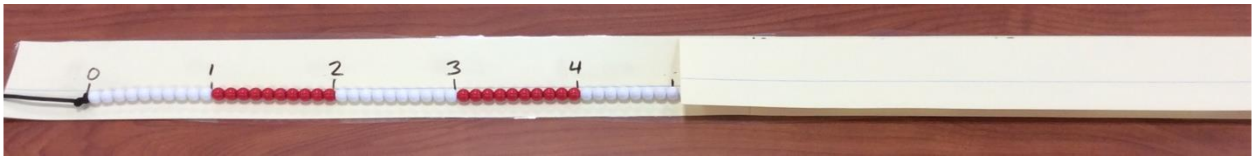
Students can use this to make connections to other representations like number lines and equations by using laminated sentence strips with the 100 bead string placed on top of the sentence strip. Students can label each group of ten on the sentence strip and then show the intervals of tens and ones they use to tell how far their given number is from 100. Then students can use their work on the sentence strip number line to write equations that match their work on their number lines.



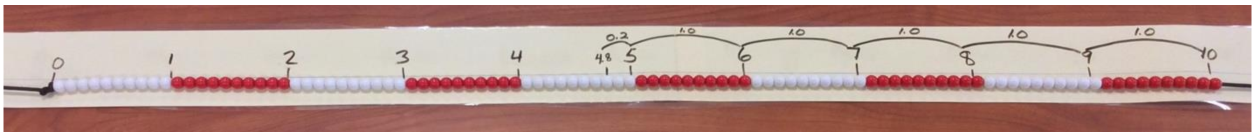
This also helps to develop making connections between addition and subtraction. Teachers can facilitate a discussion about a scenario like this one to help make the connection between addition and subtraction: “Sam says that he found how many more to 100 using counting up. Carol says she found how many more to 100 by subtracting. Who is correct? Explain.”

After students begin to show mastery of identifying how far away from 100, this can be turned into a number routine where the students use mental math to explain how many more to 100. Just flash a number represented on the 100 bead string with the remaining beads hidden and ask students how many more to 100?

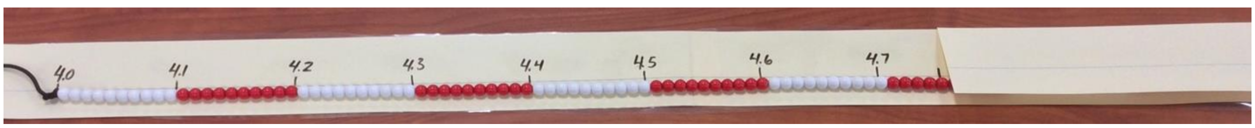
In fifth grade, these activities can be replicated with decimals. For example, if the endpoints of the 100 bead string are labeled as 0 and 10, the teacher shows part of the 100 bead string and hides or covers up the rest of the 100 bead string and asks, “How many more to 10?”

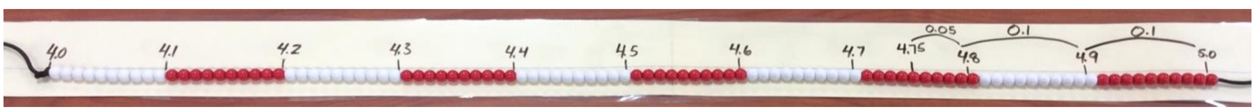


Students then use the 100 bead string to make the number and discover how many more to 10 and eventually begin making connections to number lines and equations.



In addition, this can be repeated with any interval of ten as well as intervals of one. For example, label the endpoints as 4 and 5 and ask, “How many more to 5?”

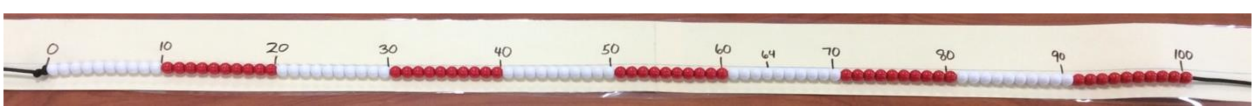




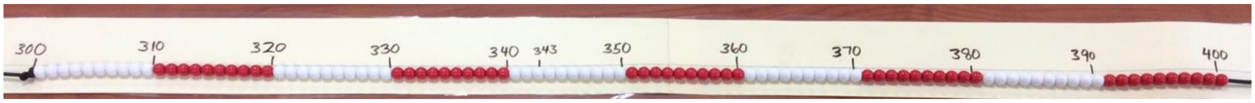
Just like with whole numbers, this can become a number routine at the beginning of math class after students demonstrate mastery with finding “how many more to x” to help students use mental math to explain how many more to the given number.

Rounding

In third grade, students round to the nearest ten and the nearest hundred. Introduce rounding by directing students to show a number from 1 – 100 on the 100 bead string. Then students can identify the tens that the number is in between. Next students can identify the ten the number rounds to by determining which ten is closer on the 100 bead string.



Next, change the endpoints to different ranges of 100, for example 200 and 300 or 700 and 800. Have students show a given number on their 100 bead string. Now students can identify the tens that the number is between and the ten that the number is closer to, and students can identify the hundreds that the number is between and the hundred that the number is closer to by using the 100 bead string to determine which ten or hundred is closer.

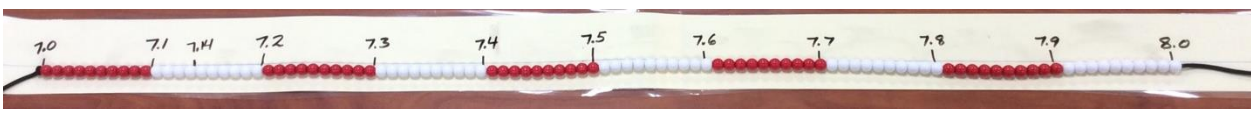


This tool is perfect for helping students to discover the "rules" for rounding as well as for defending their answers for the nearest ten and/or hundred.

Like all of the previous activities, this can also become a number routine at the beginning of math class after students have had time to develop the skill of rounding. Simply show a 100 bead string or 100 bead string image with the endpoints labeled, and ask, "What number do you see?", "What does it round to?" and "How do you know?"

In fifth grade, this can be replicated with decimals. Begin with 0 and 10 as the endpoints, have students show the decimal on their 100 bead string, identify the ones that the number is in between and then identify the nearest one.

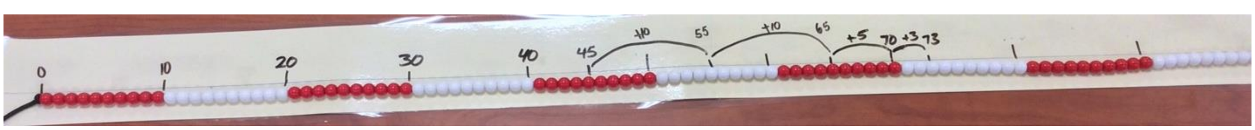
Next, the endpoints can be two whole numbers, for example 7 and 8. Students show the decimal on their 100 bead string, identify the tenths that the number is between and then identify the nearest tenth and/or whole number.



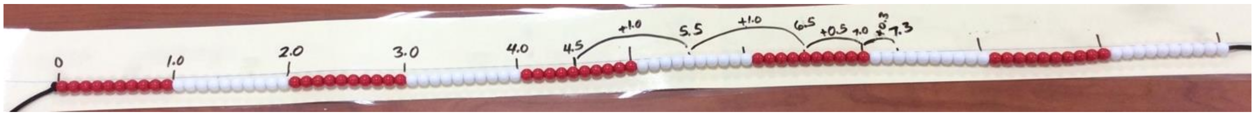
As students demonstrate mastery with identifying and rounding decimals, this can become a number routine by showing a 100 bead string or 100 bead string image with whole number endpoints labeled, and ask, "What number do you see?", "What does it round to?" and "How do you know?"

Computation

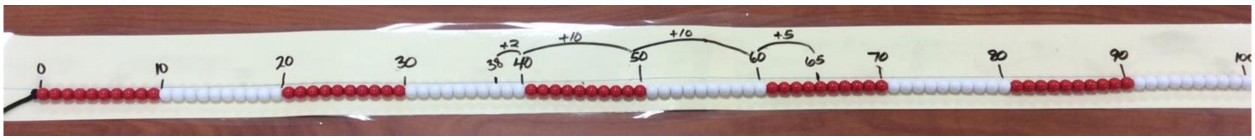
100 bead strings can also be used to develop computation strategies with whole numbers and decimals. Students can model adding up in chunks in first and second grade when adding within 100. For the expression 45 + 28 students can make 45, add two groups of ten and then a group of 8 (or a group of 5 and 3).



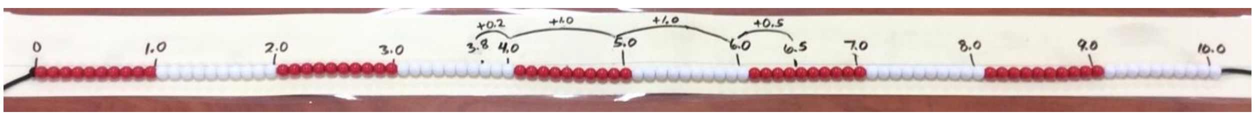
The same can be done with decimals in fifth grade. For the expression 4.5 + 2.8, students can make the number 4.5, add 2 and then add 8 tenths (or 5 tenths and 3 tenths). Teachers should also facilitate discussions to help students make connections to number lines and equations.



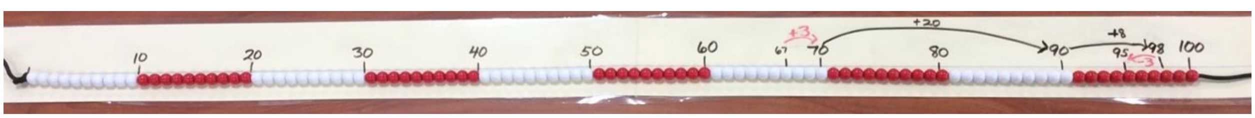
The 100 bead string can also be used with the addition strategy of making friendly numbers. For example, with 38 + 27, students can make the number 38 with their 100 bead string, add 2 to get to 40 (the friendly number) and then add 20 and then add 5 (5 is from the 7 in 27 less the 2 added to 38 to get the friendly number of 40).



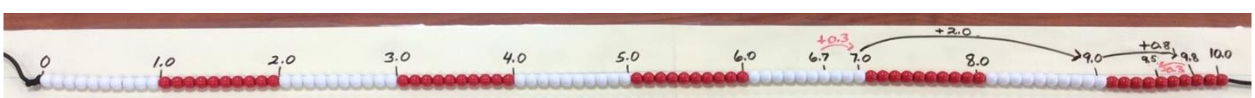
The same can be done with decimals. For the expression 3.8 + 2.7, students can make 3.8 with their 100 bead string, add 2 tenths to get to 4 (the friendly number), and then add 2 and then add 5 tenths. Again, teachers should also facilitate discussions to help students make connections to number lines and equations.



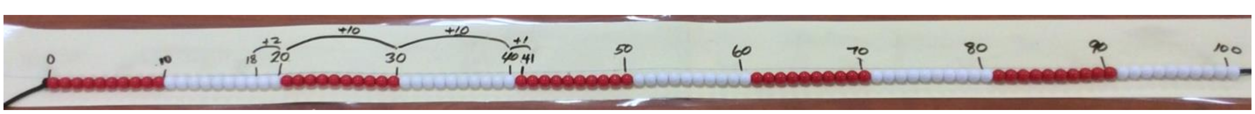
Compensation is another strategy that can be modeled with the 100 bead string. For example, with the expression 67 + 28, students can make the number 67 with their 100 bead string, add three to get to nearest ten, 70, add 28 to get to 98 and then subtract three to compensate for the adding of three to get to a friendly number to get a sum of 95.



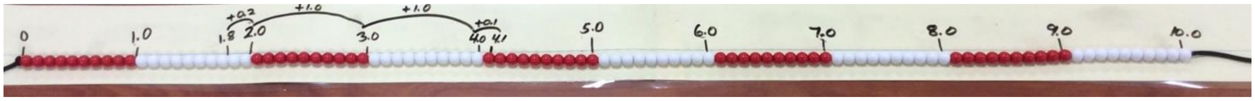
This can also be done with decimals. For example, for the expression 6.7 + 2.8, students make the number 6.7 with their 100 bead string, add 3 tenths to get to 7.0, add 2.8 to get to 9.8 and then subtract 3 tenths to get the sum of 9.5.



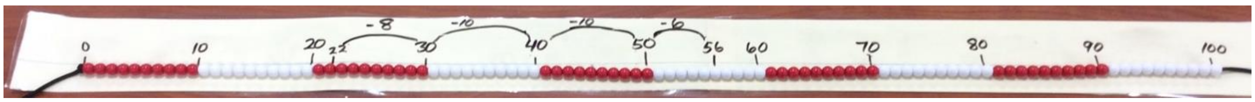
Subtraction strategies can also be modeled on the 100 bead string. Teachers and students can begin with counting up. For the expression 41 – 18, students can start with 18, count up 2 to 20, then 10 to 30, 10 more to 40 and 1 more to 41 to find the difference of 23.



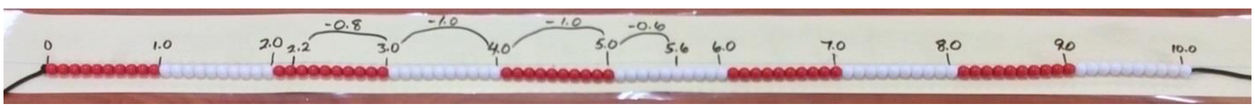
This can also be replicated with decimals. For the expression 4.1 – 1.8, students can start with 1.8, count up 0.2 to 2.0 then count up 1 to 3, 1 more to 4 and then 0.1 to 4.1 to find the difference of 2.3



Counting back (or removal) is another subtraction strategy that can be shown with the 100 bead string. For the expression 56 – 34, students start by making 56 on their 100 bead string, then subtract 6 to get to 50, 10 more to get to 40, 10 more to get to 30 and then 8 more to get to 22. Or students can start with 56, subtract 10 to get to 46, subtract 10 more to get to 36, subtract 10 more to get to 26 and then 4 to get to 22.

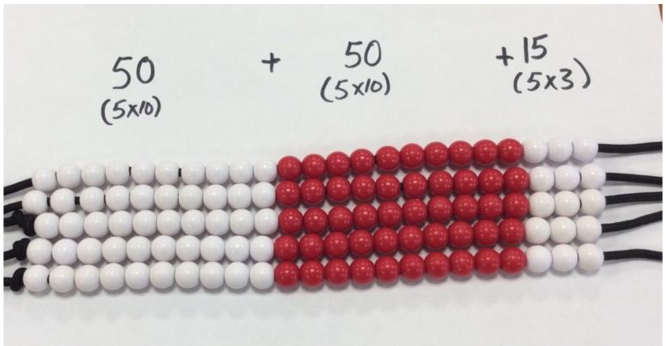


This can also be replicated with decimals. For example, for the expression 5.6 – 3.4, students can start with making 5.6 on their 100 bead string, subtract 0.6 to get to 5.0, subtract 1.0 to get to 4.0, subtract 1.0 more to get to 3.0 and then subtract 0.8 more to get to a difference of 2.2.

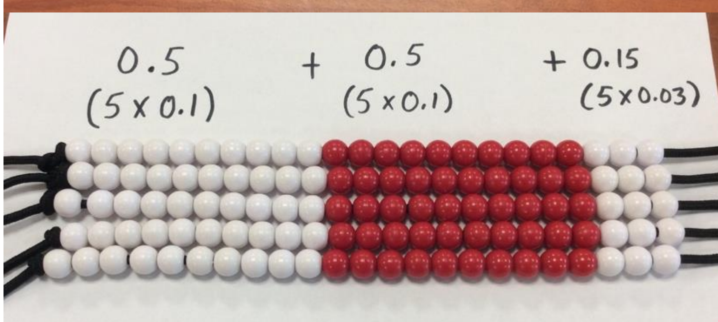


In fourth grade, the 100 bead string is also the perfect tool when beginning to teach multi-digit multiplication. For example, for the expression 5 x 23, students can make the number 23 on five 100 bead strings and line them up. Students can see 5 groups of 20 and 3 groups of 5 for a product of 115.

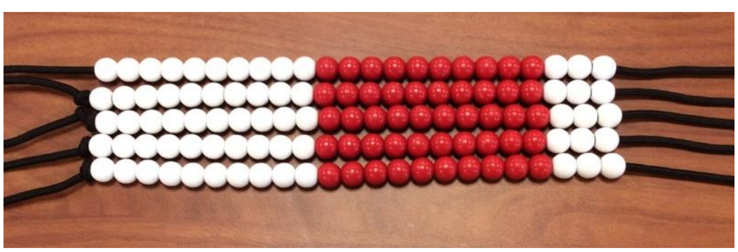
This also helps students make connections between using arrays and the area model because students can easily see the tens and ones.

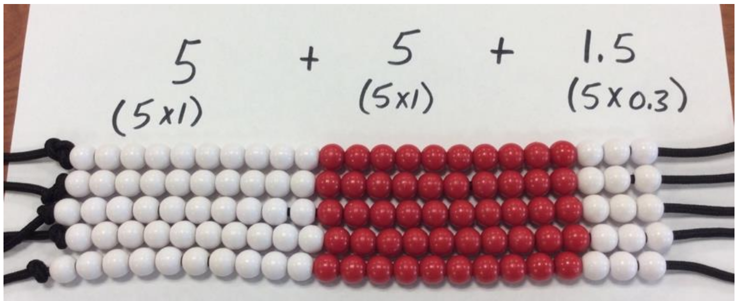


This can also be replicated when introducing multiplication with decimals in 5th grade. For example, for the expression 5 x .23, students make .23 on five 100 bead strings and line them up to see 5 groups of 2 tenths and 5 groups of 3 hundredths for a product of 1.15.



To find the answer to 5 x 2.3, students make 2.3 on five 100 bead strings and line them up to see 5 groups of 2 and 5 groups of 3 tenths for a product of 11.5.





Finally, the 100 bead string helps students make sense of the Standards for Mathematical Practice. After using the 100 bead string in class, during closure ask, "how did the 100 bead string help you to . . ." (choose 1):

- make sense of problems and persevere while solving them
- reason abstractly or quantitatively
- construct viable arguments and critique the reasoning of others
- model with mathematics
- use appropriate tools strategically
- attend to precision
- look for and make use of structure
- look for and express regularity in repeated reasoning

(NGO Center and CCSSO 2010, pp. 6–8)

While there are many more uses for developing fluency with the 100 bead string in the elementary mathematics classroom, these lesson seeds are the perfect way to start using this versatile tool. Students love using 100 bead strings and once you get started you will too!

References

National Council of Teachers of Mathematics. *Principles to Actions: Ensuring Mathematical Success for all.* Reston, VA.: NCTM, 2014

National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math). Washington, D.C.: NGA Center and CCSSO, 2010.

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